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# REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

The Reviews Section is edited by Steve Awodey (Managing Editor), John Burgess, Mark Colyvan, Anuj Dawar, Marcelo Fiore, Noam Greenberg, Rahim Moosa, Ernest Schimmerling, Carsten Schürmann, Kai Wehmeier, and Matthias Wille. Authors and publishers are requested to send, for review, copies of books to *ASL, Box 742, Vassar College, 124 Raymond Avenue, Poughkeepsie, NY 12604, USA.*

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

W. D. HART. ***The evolution of logic.*** Cambridge University Press, Cambridge, 2010, xi + 299 pp.

This book is the fifth in a series called "The Evolution of Modern Philosophy". In the introduction, it says "the series will constitute a library of modern conceptions of philosophy and will reveal how philosophy does not in fact comprise a set of timeless questions but has rather been shaped by broader intellectual and scientific developments to produce particular fields of inquiry addressing particular issues." Unfortunately, this book does not do as much as this introduction suggests to address the relevance of mathematical logic for the development of 20th century philosophy. What it does do instead is provide an introduction to four important results of mathematical logic that go beyond what most philosophers study, and develop the tools needed to prove each of these results. As a result, the book will probably be of most interest to people like me, who already have some familiarity and interest in mathematical logic, but aren't already familiar with all four of the results that are presented here. For those (primarily mathematicians) that are familiar with the results, there are a few items of historical interest in the discussion (though not as many as the title would suggest), but for those (primarily philosophers) who don't already have an interest in mathematical logic, the book doesn't do as much to motivate the results as it could.

The book largely consists of two parts. The first half, consisting of the first five chapters, outlines the development of mathematical logic with some standard historical context, from the work of Cantor and Frege to Gödel's theorems. The next four chapters provide the


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background and proofs of the "four monuments" this book is dedicated to—Gödel's proof of the consistency of the continuum hypothesis, Cohen's proof of the consistency of its negation, Friedberg and Muchnik's proof of the existence of intermediate Turing degrees, and Morley's proof that categoricity in one uncountable cardinality entails categoricity in all of them. The last chapter provides a sort of summary, and an attempt to motivate the philosophical interest of these four results, and more importantly, the methods that lead to them.

It is hard to tell what level of familiarity with mathematical logic is assumed in the reader of this book. The proofs of the "four monuments" become quite intricate, and will thus be hard to follow for readers that are not mathematically sophisticated. But all relevant background is included, down to the level of set notation, and the definition of an ordered pair, so that a devoted student could theoretically learn all the material from this book.

However, the writing style will be quite intimidating for those that aren't already familiar with much of the material. This is especially true because there are no sections within chapters, no headings indicating new topics, and in general no easy way to flip back a page or two to find out what some letter stands for in the current or previous proof. In textbooks this is often less of a problem, because tools are introduced in order of complexity, with well-delineated sections. But in this book, techniques are introduced as they are needed—Tarski's theory of truth is introduced as part of a digression in discussing the analytic/synthetic distinction in chapter 2, and the notions of topology are introduced in the middle of some extended discussion of model theory in chapter 9. When Hart begins a proof or a discussion of some philosophical concept, the reader doesn't know how long it will take, and how many other topics and discussions will be introduced on the way.

In the first half of the book, chapters 1 and 5 are dedicated primarily to mathematical technicalities (Cantorian set theory, and Gödel's theorems, respectively), while chapters 2, 3, and 4 are devoted more to the historical and philosophical issues that accompany them. Chapter 2 follows Kant's notions of analyticity and a priority, and shows how they developed in the "low-water mark in the history of logic" (p. 35), followed by Frege's attempt to redevelop them in terms of some notion of the dependence of some truths on others. Chapter 3 discusses the debate between Russell and Poincaré about the vicious-circle principle, the development of stratified type theory, and the contrast with Quine's "New Foundations". Chapter 4 discusses "Orayen's paradox"—the fact that Tarski's results seem to show that truth and quantification only make sense over a set-sized domain, and yet we seem to be able to assert some things with full generality. I think that these chapters could have used more contact with some contemporary literature of the past two decades (for instance, recent work by metaphysicians on notions of dependence and grounding, the use of various new non-classical logics for a hierarchy-free theory of truth, and the anglophone philosophical discussion of absolute generality—there is some interesting discussion of the Spanish language literature on this topic, and it would be interesting for the two literatures to engage one another) but for mathematicians, they will still be an interesting introduction to some philosophical topics that are connected to the mathematics.

In chapters 6 through 9 on the other hand, there is very little attempt to connect the mathematics to philosophical topics. There is some suggestion that the way the mathematical techniques are used leads us to platonist intuitions, but no strong argument for this view. In the chapter on recursion theory and Turing degrees, there is a discussion of the Kantian study of reason, which we must understand in terms of deductive reasoning, which should therefore lead us to consider the complexity of various deductive theories. But it is never clear how the study of Turing degrees (which is definitely very interesting) will shed light on any of the interesting philosophical questions.

The mathematical exposition of these results is not original (the two results in set theory follow Hilary Putnam's lecture notes from 1968, while the discussions of recursion theory and model theory follow the classic textbooks by Rogers and Marker, respectively). So the main advantage of this book is to have these four results in one place, with all of the facts that they depend on included as well. This book would be much more useful as a reference if these chapters were structured more like textbooks, but even as they stand they still suffice to learn the material. Although I was already familiar with the set-theoretic results, I found the method used to be slightly less readable than either the expositions of forcing used in the books by Kunen and Jech, or the method of Boolean-valued models introduced by Dana Scott and taught in the book by Bell. But the results from recursion theory and model theory were easier to follow, despite the fact that I was less familiar with them.

Overall, I think that the mathematics would have been more helpful if it had been more clearly organized, and the philosophy would have been more useful if it had more up-to-date references rather than primarily citing the classic discussions of Quine, Putnam, Grice, and their contemporaries. But I found that the book provided a useful way for me to learn central results of areas of mathematical logic that I wasn't as familiar with, and it provided me with a few new historical insights (for instance, according to a footnote on p. 257, Ramsey initially developed Ramsey theory in an attempt to find an algorithm for deciding validity in first-order logic—it's interesting to see how such a fruitful mathematical theory came from a doomed project of this sort).

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WILLIAM J. MITCHELL. *$I[\omega_2]$ can be the nonstationary ideal on $\mathsf{Cof}(\omega_1)$*. ***Transactions of the American Mathematical Society***, vol. 361 (2009), no. 2, pp. 561–601.

This influential paper brings an important breakthrough on the long-standing open question if Shelah's approachability ideal $I[\omega_2]$ can be trivial on $\omega_1$-cofinal ordinals, giving an affirmative answer. Starting from a model with a cardinal $\kappa$ that is $\kappa^+$-Mahlo, the author introduces a forcing notion that is proper, turns $\kappa$ into $\omega_2$ and arranges that no stationary subset of $\omega_2 \cap \mathsf{cof}(\omega_1)$ is in $I[\omega_2]$. The statement that $\kappa$ is $\kappa^+$-Mahlo is a large cardinal concept somewhat stronger than Mahloness, but well below weak compactness. This large cardinal assumption is necessary for the result as was shown previously by Shelah; the argument can be found in Mitchell's paper *A weak variation on Shelah's $I[\omega_2]$*, ***The Journal of Symbolic Logic***, vol. 69 (2004), pp. 94–100.

Given a regular cardinal $\lambda$ and a sequence $\vec{a} = \langle a_\alpha \mid \alpha < \lambda \rangle$ the set $B(\vec{a})$ consists of those $\nu < \lambda$ for which there is a set $c \subseteq \nu$ whose order type agrees with the cofinality of $\nu$ and all proper initial segments of $c$ are among $a_\alpha$ for $\alpha < \nu$. $I[\lambda]$ is the ideal of subsets of $\lambda$ generated by nonstationary sets and sets of the form $B(\vec{a})$. Shelah introduced the ideal $I[\lambda]$ in the appendix on stationary sets to his *Classification theory for non-elementary classes*, ***Lecture Notes in Mathematics***, vol. 1292, pp. 483–497. In *Reflecting stationary sets and successors of singular cardinals*, ***Archive for Mathematical Logic***, vol. 31 (1991), pp. 25–53 he discusses in detail its basic properties; in particular he establishes the consistency result that for regular $\kappa$ the ideal $I[\kappa^+] \cap \mathsf{cof}(\kappa)$ is generated over the nonstationary ideal by a single stationary-costationary set. Finally he makes a substantial use of the ideal in the construction of a model that satisfies full stationary reflection along with the maximum possible amount of supercompactness. There, too, Shelah asks the question if it is consistent that $I[\kappa^+] \cap \mathsf{cof}(\kappa)$ is the nonstationary ideal.

The main ingredient in Mitchell's construction is a new method of adding a closed unbounded subset of $\kappa$ while simultaneously collapsing all cardinals between $\omega_1$ and $\kappa$ to $\omega_1$ via a forcing notion consisting of finite conditions, so that $\kappa$ becomes $\omega_2$ in the generic extension. Adding closed unbounded subsets is a natural attempt to destroy all stationary sets that are in $I[\omega_2]$. The use of finite conditions is motivated by the fact that standard forcings for